Parallelizing Neural Network Models Effectively on GPU by Implementing Reductions Atomically

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Outline

Introduction

- 2 Dimension Flattening
- 3 Polyhedral Loop Transformations
- 4 Code Generation and Optimization
- **5** Experimental Results
- 6 Conclusion

Definition

Reduction is a binary operator \circledast that applies to each element of an input vector V and reduces V to a single value r. Formally,

$$r = \begin{pmatrix} {}^{\circledast_{i=1}^{n} (v_{i}^{(1)})} \\ \vdots \\ {}^{\circledast_{i=1}^{n} (v_{i}^{(d)})} \end{pmatrix} = \begin{pmatrix} {}^{v_{1}^{(1)} \circledast v_{2}^{(1)} \circledast \cdots \circledast v_{n}^{(1)}} \\ \vdots \\ {}^{v_{1}^{(d)} \circledast v_{2}^{(d)} \circledast \cdots \circledast v_{n}^{(d)}} \end{pmatrix}$$

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- Reduction is involved in less compute-intensive operators (e.g., SoftMax, ReLU, BachNorm) of neural network models.
- Ineffective parallelization of reductions can hamper the performance of such operators, which can in turn result in sub-optimal performance.
- Optimizing reduction is thus important for parallelizing neural network models but not well studied before.

Introduction

Background

Why do we target GPU



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- Parallelism in reduction makes GPU an attractive and suitable target.
- GPU abstracts the streaming multiprocessors as blocks and CUDA cores as threads. The number of threads within a block is limited.





• Parallel Reductions on GPU

• Polyhedral Parallel Reductions



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 - $\bullet~{\sf Harris}^{[1]}$ revealed many optimization useful for library-based methods

• Polyhedral Parallel Reductions

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interleaved addressing



sequential addressing

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 - Polyhedral compilation^[2] easily composes loop transformations

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 - Polyhedral compilation^[2] easily composes loop transformations
 - Wastes GPU resources when handling multiple, small reduction dims
 - Ineffective handling of global synchronization through privatization

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(a) Reductions over all loop dimensions; (b) and (c) Both the (red) parallel dimensions and the (blue) reduced dimensions are continuous; (d) The parallel dimensions and the reduced dimensions are interleaved.

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• (a) can be flattened into all-reduce

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\begin{array}{c|c} \mbox{reduced for } j=0 \ \mbox{to } \mathbb{N} \\ R(j_1,\cdots,j_r); \\ a/l\mbox{reduce.} \end{array} \qquad \begin{array}{c} \mbox{parallel for } i=0 \ \mbox{to } \mathbb{N} \\ \mbox{reduced for } j=0 \ \mbox{to } \mathbb{N} \\ \mbox{reduced for } j=0 \ \mbox{to } \mathbb{N} \\ \mbox{reduced for } j=0 \ \mbox{to } \mathbb{N} \\ \mbox{reduced for } j=0 \ \mbox{to } \mathbb{N} \\ \mbox{reduced for } j=0 \ \mbox{to } \mathbb{N} \\ \mbox{R(}i_1,\cdots,i_p,j_1,\cdots,j_r); \\ \mbox{R(}i_1,\cdots,i_p,j_1,\cdots,j_r); \\ \mbox{R(}i_1,\cdots,i_p,j_1,\cdots,i_p); \\ \mbox{reduce.} \\ \mbox{Y-reduce.} \end{array}
```

Dimension Flattening

Loop Coalescing

October 12, Chicago, Illinois 7 / 23

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- Nested reductions over multiple variables are frequent
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(a) Reductions over all loop dimensions; (b) and (c) Both the (red) parallel dimensions and the (blue) reduced dimensions are continuous; (d) The parallel dimensions and the reduced dimensions are interleaved.

• (a) can be flattened into *all*-reduce; (b) and (c) can be flattened into *x*- and *y*-reduce; (d) needs loop interchange

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October 12, Chicago, Illinois 7 / 23

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```
\begin{array}{c|c} \mbox{reduced for } j=0 \ \mbox{to N} \\ R(j_1,\cdots,j_r); \\ all\mbox{-reduce.} \end{array} \begin{array}{c|c} \mbox{parallel for } i=0 \ \mbox{to M} \\ reduced \ \mbox{for } j=0 \ \mbox{to N} \\ R(i_1,\cdots,i_p,j_1,\cdots,j_r); \\ reduced \ \mbox{areallel for } i=0 \ \mbox{to M} \\ R(j_1,\cdots,j_r,i_1,\cdots,i_p); \\ R(j_1,\cdots,j_r,i_1,\cdots,i_p); \\ R(j_1,\cdots,j_r,i_1,\cdots,i_p); \\ \mbox{areallel for } i=0 \ \mbox{to M} \\ R(j_1,\cdots,j_r,i_1,\cdots,i_p); \\ \mbox{areallel for } i=0 \ \mbox{to M} \\ R(j_1,\cdots,j_r,i_1,\cdots,i_p); \\ \mbox{areallel for } i=0 \ \mbox{to M} \\ R(j_1,\cdots,j_r,i_1,\cdots,i_p); \\ \mbox{areallel for } i=0 \ \mbox{to M} \\ \mbox{areallel for } i=0 \ \mbox{areallel for i=0 \ \m
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Dimension Flattening

Loop Coalescing

October 12, Chicago, Illinois 7 / 23

• Transformation formula of tensor indexes

$$\begin{cases} M = \prod_{x=1}^{p} s_x = s_1 \times \cdots \times s_p, N = \prod_{y=1}^{r} t_y = t_1 \times \cdots \times t_r; \\ i_a = \left\lfloor i \middle/ \prod_{x=a+1}^{p} s_x \right\rfloor \mod s_a : (1 \le a < p), i_p = i \mod s_p; \\ j_b = \left\lfloor j \middle/ \prod_{y=b+1}^{r} t_y \right\rfloor \mod t_b : (1 \le b < r), j_r = j \mod t_r; \end{cases}$$

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- At most one s_x and one t_y can be symbolic constants, making the flattend dimensions amenable to polyhedral complication
- An example of dimension flattening

```
for w=0 to 20
                                                      for h=0 to 40
   for x=0 to 10
                                                        for w=0 to 20
      for y=0 to 5
                                                          for x=0 to 10
        E(h.w.x.v):
                                                            for y=0 to 5
for h=0 to 40
                                                              E(h,w,x,y);
 for w=0 to 20
                                                      for i=0 to 20
   for x=0 to 10
                                                        for i=0 to 40*10*5
      for v=0 to 5
                                                          R(i,(j/(10*5))%40,(j/5)%10,j%5);
        R(\underline{h}, w, \underline{x}, y);
                                                                      < □ > < □ > < □ > < □ > < □ > < □ >
```

for h=0 to 40

Propagating reduction dependences to enable fusion

Loop coalescing invalidates the originally possible fusion



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Loop coalescing invalidates the originally possible fusion



Propagate the dependences along the reduced dims

```
for h=0 to 40
  for w=0 to 20
  for x=0 to 10
    for y=0 to 5
    E(h,w,x,y);
  for h=0 to 40
  for w=0 to 20
    for x=0 to 10
    for y=0 to 5
    R(h,w,x,y);
```

```
for h=0 to 40
for w=0 to 20
for x=0 to 20
for x=0 to 10
for y=0 to 5
for y=0 to 5
for y=0 to 5
for y=0 to 20
for i=0 to 20
for i=0 to 20
for i=0 to 40*10*5
for j=0 to 40*10*5
R(i,(j/(10*5))%40,(j/5)%10,j%5);
R(i,(j/(10*5))%40,(j/5)%10,j%5);
R(i,(j/(10*5))%40,(j/5)%10,j%5);
```

(I) < (II) <

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for x=0 to 10
for y=0 to 5
R(h,w,x,y);
```

```
for h=0 to 40
for w=0 to 20
    for x=0 to 20
    for x=0 to 10
    for y=0 to 5
    for y=0 to 5
    for i=0 to 20
for i=0 to 20
for i=0 to 20
for i=0 to 40*10*5
    R(i,(j/(10*5))%40,(j/5)%10,j%5);
     R(i,(j/(10*5))%40,(j/5)%10,j%5);
    R(i,(j/(10*5))%10,j%5);
    R(i,(j/(10*5))%10,j\%5);
    R(i,(
```

• Recover the fusion possibility

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Propagating reduction dependences to enable fusion

• Loop coalescing invalidates the originally possible fusion



• Propagate the dependences along the reduced dims

```
for h=0 to 40
 for w=0 to 20
                                for h=0 to 40
   for x=0 to 10
                                 for w=0 to 20
                                                                   parallel for i=0 to 20
     for v=0 to 5
                                   for x=0 to 10
                                                                     parallel for j=0 to 40*10*5
       E(h.w.x.v):
                                     for y=0 to 5
                                                                       E(i,(j/(10*5))%40,(j/5)%10,j%5);
for h=0 to 40
                                       E(h.w.x.v):
                                                                   parallel for i=0 to 20
 for w=0 to 20
                                for i=0 to 20
                                                                     reduced for j=0 to 40*10*5
   for x=0 to 10
                                 for j=0 to 40*10*5
                                                                       R(i,(j/(10*5))%40,(j/5)%10,j%5);
     for y=0 to 5
                                   R(i,(i/(10*5))%40,(i/5)%10,i%5):
       R(h,w,x,y);
```

- Recover the fusion possibility
- Fusion with follow-up elementwise operators is handled by Apollo^[1]

^[1] Jie Zhao et al. "Apollo: Automatic Partition-based Operator Fusion through Layer by Layer Optimization". Vol. 4. MLSys'22. 2022, pp. 1–19.

Dimension Flattening

Reduction Propagation

• Transformation formula of tensor indexes guarantees the "static affine control" requirement of polyhedral compilation

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- Polyhedral loop fusion is the default heuristic of *isl*^[1], reinforced by the post-tiling fusion strategy^[2] embedded in AKG when necessary

^[1]Sven Verdoolaege. "IsI: An Integer Set Library for the Polyhedral Model". ICMS'10, pp. 299–302.

^[2] Jie Zhao et al. "Optimizing the Memory Hierarchy by Compositing Automatic Transformations on Computations and Data". MICRO'20, pp. 427-441.

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- Always guarantee outer parallelism (possibly by converting a *y*-reduce into an *x*-reduce pattern)
- bind a parallel loop to outer GPU block dims and a reduced loop to inner

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^[2] Jie Zhao et al. "Optimizing the Memory Hierarchy by Compositing Automatic Transformations on Computations and Data". MICRO'20, pp. 427–441.

• Tiling is performed on top of a fusion configuration

```
/* Tile sizes are 32 × 4. */
parallel for ib=0 to M/32
reduced for jb=0 to N/4
parallel for it=0 to 32
reduced for jt=0 to 4
    m elmwise stmts;
    // marked reduce stmt
    R(i1,...,ip,j1,...,jr);
```

The tiled code.

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• Tiling is performed on top of a fusion configuration



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Tiling and binding reduced dimensions

• Tiling is performed on top of a fusion configuration



• The outer parallel loops can be bound safely

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Tiling and binding reduced dimensions

• Tiling is performed on top of a fusion configuration



The tiled code.



Hardware binding.

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- The outer parallel loops can be bound safely
- The inner reduced loops are bound by ignoring reduction dependences

Tiling and binding reduced dimensions

Tiling is performed on top of a fusion configuration



The tiled code.

Hardware binding.

parallel

- The outer parallel loops can be bound safely
- The inner reduced loops are bound by ignoring reduction dependences
- This enables the possibility to decompose a reduction operator across multiple blocks when handling large reduction dimensions

.

blockIdx.v

threadIdx.v

• Tiling is performed on top of a fusion configuration



The tiled code.



Hardware binding.

- The outer parallel loops can be bound safely
- The inner reduced loops are bound by ignoring reduction dependences
- This enables the possibility to decompose a reduction operator across multiple blocks when handling large reduction dimensions
- Ignored dependences will be resumed during code generation

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• Loop coalescing is an achievable but undesired transformation in polyhedral compilation^[1], we isolates it as a preprocessing step in dimension flattening.

[1] Sven Verdoolaege et al. "Scheduling for PPCG". Report CW 706 (2017). $\langle \Box \rangle \langle \Box \rangle \langle \Xi \rangle$

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- Isolating loop coalescing also makes it possible to canonicalize reduction patterns, as shown before.

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- Loop interchange before the polyhedral transformations can be harmful to memory coalescing; we avoid this risk by reasoning about tensor layouts using tensor expression language.
- Isolating loop coalescing also makes it possible to canonicalize reduction patterns, as shown before.
- Our three canonical reduction forms simplify hardware binding strategies and compress the search space of tile sizes.





• Part ① enables sequential addressing and fusion with other operators



- Part 1 enables sequential addressing and fusion with other operators
- Part ② ensures higher performance than stand-alone compilation approaches^{[1][2]} and minimizes the number of involved blocks

Code Generation and Optimization

^[1]Tianqi Chen et al. "TVM: An Automated End-to-End Optimizing Compiler for Deep Learning". OSDI 2018, pp. 578–594.

^[2] Jie Zhao et al. "AKG: Automatic Kernel Generation for Neural Processing Units Using Polyhedral Transformations". PLDI 2021, pp. 1233–1248.



- Part (1) enables sequential addressing and fusion with other operators
- Part ② ensures higher performance than stand-alone compilation approaches^{[1][2]} and minimizes the number of involved blocks
- Part ③ carries out global synchronization using atomic instructions, avoiding the need to invoke multiple kernels for neural network models

[1] Tianqi Chen et al. "TVM: An Automated End-to-End Optimizing Compiler for Deep Learning". OSDI 2018, pp. 578–594.
 [2] Jie Zhao et al. "AKG: Automatic Kernel Generation for Neural Processing Units Using Polyhedral Transformations". PLDI 2021, pp. 1233–1248.

Code Generation and Optimization

Self-developed Library

Example templated code

```
__global__ void reduce(int len, T *input, T *output, int num, OP op){
 T local sum=0:
 __shared__ T shared_buf[4];
 __shared__ T block_sum[1];
 /* Part 1, automatically generated using polyhedral compilation. */
 for(int k=0; k< num: k++)</pre>
   if(threadIdx.x+k*blockDim.x+blockIdx.x*blockDim.x*num<len)
     op(local_sum,input[threadIdx.x+k*blockDim.x+blockIdx.x*blockDim.x*num]);
 synchthreads():
 /* Part 2, automatic invocation of library routines. */
 Parallel_Reduce<T, OP, 4, all>(op, &block_sum[0], shared_buf, local_sum);
 synchthreads():
 /* Part 3, automatic global sychronization using atomics. */
 if(threadIdx.x==0)
   Atomic_Return<T, OP>(block_sum[0],&output[0],op);
}
```

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Example templated code

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__global__ void reduce(int len, T *input, T *output, int num, OP op){
 T local sum=0:
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 /* Part 1, automatically generated using polyhedral compilation. */
 for(int k=0; k< num; k++)</pre>
   if(threadIdx.x+k*blockDim.x+blockIdx.x*blockDim.x*num<len)
     op(local_sum,input[threadIdx.x+k*blockDim.x+blockIdx.x*blockDim.x*num]);
 synchthreads():
 /* Part 2, automatic invocation of library routines. */
 Parallel_Reduce<T, OP, 4, all>(op, &block_sum[0], shared_buf, local_sum);
 synchthreads():
 /* Part 3, automatic global sychronization using atomics. */
 if(threadIdx.x==0)
   Atomic_Return<T, OP>(block_sum[0],&output[0],op);
}
```

- OP can be instanced using *summation*, *product*, *min*, *max*, *logical* AND and *logical* OR
- T can be one of double, float32, float16, bool, long long int and int
- Parallel_Reduce and Atomic_Return are interfaces to our library and low-level atomic instructions
- __synchthreads() is automatically inserted

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• Handling an irregular input size n

- Handling an irregular input size n
 - Divide *n* into 2^k (greatest power of two less than *n*) and $n 2^k$
 - Perform a local reduction to convert n into an irregular size 2^k



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• Generalization to multiple reductions

- Handling an irregular input size n
 - Divide n into 2^k (greatest power of two less than n) and $n 2^k$
 - Perform a local reduction to convert n into an irregular size 2^k
 - Designed for Part ②, this optimization is also useful for irregular sizes across multiple blocks



- Generalization to multiple reductions
 - Still fusible when their enclosing loop nests are identical and reductions patterns are the same
 - More complicated scenarios can be feedback to the upstream graph compiler^[1] for exploiting more fusion opportunities

^[1] Jie Zhao et al. "Apollo: Automatic Partition-based Operator Fusion through Layer by Layer Optimization". Vol. 4. MLSys'22. 2022, pp. 1–19.

Example code for fused reductions

```
__global__ void reduce(float *input0, float *input1, float *input2, float *output0, float *output1){
 float local sum=0: float local max=-3.40282e+38f;
 __shared__ float shared_buf[128]; __shared__ float block_sum[1];
 __shared__ float block_max[1];
 /* Fuse the addition operator with reduce_sum. */
 for(int k=0; k< 8; k++)</pre>
   if(threadIdx.x+k*blockDim.x+blockIdx.x*blockDim.x*8<1024){
     float agg_local = input0[threadIdx.x+k*blockDim.x+blockIdx.x*blockDim.x*8]
       + input1[threadIdx.x+k*blockDim.x+blockIdx.x*blockDim.x*8];
     Sum(local_sum, agg_local);
   ŀ
 synchthreads():
 Parallel_Reduce<float,Sum,128,all>(Sum,&block_sum[0],shared_buf,local_sum);
 __synchthreads();
 if(threadIdx.x==0)
   output0[0] = block_sum[0];
 __synchthreads();
 /* Fuse two reductions through identical hardware configuration. */
 for(int k=0: k< 17: k++)</pre>
   if(threadIdx.x+k*blockDim.x+blockIdx.x*blockDim.x*17 < 2176)
     Max(local_max, input2[threadIdx.x+k*blockDim.x+blockIdx.x*blockDim.x*17]);
 __synchthreads();
 Parallel_Reduce<float,Max,128,all>(Max,&block_max[0],shared_buf,local_max);
 __synchthreads();
 if(threadIdx.x==0)
   output1[0] = block max[0];
3
```

Fusing one addition and two reductions. It first sums input0 and input1, both of which are 1D tensors of size 1024, and outputs output0 through a *reduce_sum*. Another 1D tensor input2 of size 2176 is reduced (*reduced_max*) to output1.

Hardware	NIVDIA Tesla V100 GPU			
Operating system	Ubuntu 16.04.4 LTS (GNU/Linux			
	4.4.0-116-generic x86_64)			
CUDA toolkit	version 10.1, -O3 option			
Python	version 3.7.5			
Neural network framework	MindSpore ^[1] , version 1.8.1			
Baselines	AKG ^[2] , TVM (v0.6) ^[3] , Ansor ^[4] ,			
	cuDNN (v7.6.4) and CUB (v1.8) ^[5]			
Reported time	Geometric mean of 10 executions			

[4] Lianmin Zheng et al. "Ansor: Generating High-Performance Tensor Programs for Deep Learning". OSDI 2020, pp. 863–879.

[5] Nvidia. CUB Documentation. 2018. URL: https://nvlabs.github.io/cub/. < D > < B > < E > < E > < E >

Experimental Results

^[1]Huawei. MindSpore. 2020. URL: https://www.mindspore.cn/en.

^[2] Jie Zhao et al. "AKG: Automatic Kernel Generation for Neural Processing Units Using Polyhedral Transformations". PLDI 2021, pp. 1233–1248.

^[3]Tianqi Chen et al. "TVM: An Automated End-to-End Optimizing Compiler for Deep Learning". OSDI 2018, pp. 578–594.



- 4 data types: *float*32, *float*16, *int* and *bool*
- 10 different input shape configurations (*reduce_sum x* axis: original shapes; *reduce_max x* axis: flattened shapes)
- y axis: log scaled execution time in microseconds; lower is better



- 3 operators: reduce_sum, reduce_max and reduce_and
- 4 data types: float32, float16, int and bool •
- 10 different input shape configurations (reduce_sum x axis: original shapes; *reduce_max x* axis: flattened shapes)
- y axis: log scaled execution time in microseconds; lower is better
- Please refer to the paper for the result of reduce_and



- Ansor sometimes quits when handling y-reduce
- $\bullet\,$ cuDNN may not perform loop coalescing and always uses an identical 3D thread configuration << 8, 16, 1>>
- CUB seems more suitable for reductions along the inner loops



- Ansor sometimes quits when handling y-reduce
- $\bullet\,$ cuDNN may not perform loop coalescing and always uses an identical 3D thread configuration << 8, 16, 1>>
- CUB seems more suitable for reductions along the inner loops
- PANAMERA outperforms cuDNN, CUB, TVM and Ansor by 33.7×, 3.5×, 5.4× and 9.6×, respectively

Experimental Results

Results of Single Operators

Summary of fused operators. cast16 converts an *f*32 tensor into *f*16 and cast32 performs the reverse process; r_sum represents the reduce_sum operator.

no.	input config.	op_1	op ₂	op ₃	op ₄	op ₅	<i>op</i> ₆
1	f32 [<u>64</u> ,2]	cast16	cast32	cast16	r_sum	-	-
2	f32 [<u>1280</u> ,21128]	cast16	r_sum	-	-	-	-
3	<i>f</i> 16 [<u>64</u> ,768]	cast32	r_sum	-	-	-	-
4	f32 [1280, <u>21128]</u>	mul	r_sum	-	-	-	-
5	f32 [<u>1280]</u>	neg	mul	r_sum	-	-	-
6	f32 [<u>3072]</u>	mul	mul	r_sum	-	-	-
7	f32 [<u>64</u> ,128,768]	add	mul	_sum	-	-	-
8	f32 [<u>64,128</u> ,768]	add	mul	r_sum	add	mul	r_sum
9	f32 [<u>8192</u> ,768]	r_sum	r_sum	-	-	-	-
10	f16 [<u>64,128</u> ,12,64]	reshape	cast32	r_sum	-	-	-
11	f16 [<u>64,128</u> ,768]	reshape	cast32	r_sum	-	-	-
12	f16 [<u>64,20</u>]	reshape	r_sum	-	-	-	-

Results of fused operators



- While not exploiting fusion, cuDNN does not support *type casting* or *reshaping* operators
- TVM/Ansor under-performs when multi-block parallelism is essential for performance

Results of fused operators



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Results of fused operators



- While not exploiting fusion, cuDNN does not support *type casting* or *reshaping* operators; PANAMERA exhibits a better scalablity by handling more scenarios
- TVM/Ansor under-performs when multi-block parallelism is essential for performance; PANAMERA can better handle the multi-block parallelism
- On average, PANAMERA outperforms cuDNN, TVM and Ansor by 9.5×, 2.6× and 2.7×, respectively

Experimental Results

Results of Fused Operators

Execution time in milliseconds (GPT-3 is executed on a Tesla A100 GPU)

Workloade	MindSpore	T\/M	Ancor	AKG PANAMERA Improve	Improveme	ent over	number of			
WORKIOAUS	windspore	1 1 1 1	Alisor	ANG	IANAMERA	MindSpore	TVM	Ansor	AKG	fused ops
BERT ^[1]	352.2	138.0	120.3	124.0	111.0	+217%	+24%	+8%	+12%	304
Wide&Deep ^[2]	22.4	12.5	12.8	12.6	11.0	+104%	+14%	+16%	+15%	74
VGG-16 ^[3]	70.4	65.7	66.3	67.6	64.2	+10%	+2%	+3%	+5%	39
MobileNet-v3 ^[4]	151.4	133.0	129.4	136.8	131.5	+15%	+1%	-2%	+4%	52
Transformer ^[5]	157.8	132.4	126.5	136.8	79.2	+99%	+67%	+60%	+73%	746
GPT-3 ^[6]	483.0	133.9	131.3	146.2	123.7	+290%	+8%	+6%	+18%	409
average						+122.5%	+19.3%	+15.2%	+21.2%	

^[2]Heng-Tze Cheng et al. "Wide & Deep Learning for Recommender Systems". DLRS 2016, pp. 7-10.

^[3]Karen Simonyan et al. Very Deep Convolutional Networks for Large-Scale Image Recognition. 2014. arXiv: 1409.1556 [cs.CV].

^[4]Andrew G. Howard et al. MobileNets: Efficient Convolutional Neural Networks for Mobile Vision Applications. 2017. arXiv: 1704.04861 [cs.CV].

^[5]Ashish Vaswani et al. "Attention is All You Need". NIPS'17, pp. 6000-6010.

^[6]Tom Brown et al. "Language Models are Few-Shot Learners". NeurIPS 2020, pp. ±877–1991 🖉 🗧 🛌 🛬 👘 🛬 👘

Experimental Results

Results of End-to-end Workloads

^[1] Jacob Devlin et al. "BERT: Pre-training of Deep Bidirectional Transformers for Language Understanding". NAACL 2019, Volumn 1, pp. 4171–4186.

Execution time in milliseconds (GPT-3 is executed on a Tesla A100 GPU)

Workloads	MindSpore	TVM	Ancor	AKC	DANAMEDA	Improvement over				number of
WORKIDads	windspore	1 1 1 1	Alisor	ANG	IANAMERA	MindSpore	TVM	Ansor	AKG	fused ops
BERT ^[1]	352.2	138.0	120.3	124.0	111.0	+217%	+24%	+8%	+12%	304
Wide&Deep ^[2]	22.4	12.5	12.8	12.6	11.0	+104%	+14%	+16%	+15%	74
VGG-16 ^[3]	70.4	65.7	66.3	67.6	64.2	+10%	+2%	+3%	+5%	39
MobileNet-v3 ^[4]	151.4	133.0	129.4	136.8	131.5	+15%	+1%	-2%	+4%	52
Transformer ^[5]	157.8	132.4	126.5	136.8	79.2	+99%	+67%	+60%	+73%	746
GPT-3 ^[6]	483.0	133.9	131.3	146.2	123.7	+290%	+8%	+6%	+18%	409
average						+122.5%	+19.3%	+15.2%	+21.2%	

• number of operators fused by PANAMERA in a workload

^[2]Heng-Tze Cheng et al. "Wide & Deep Learning for Recommender Systems". DLRS 2016, pp. 7-10.

^[5]Ashish Vaswani et al. "Attention is All You Need". NIPS'17, pp. 6000-6010.

Experimental Results

^[1] Jacob Devlin et al. "BERT: Pre-training of Deep Bidirectional Transformers for Language Understanding". NAACL 2019, Volumn 1, pp. 4171–4186.

^[3]Karen Simonyan et al. Very Deep Convolutional Networks for Large-Scale Image Recognition. 2014. arXiv: 1409.1556 [cs.CV].

^[4]Andrew G. Howard et al. MobileNets: Efficient Convolutional Neural Networks for Mobile Vision Applications. 2017. arXiv: 1704.04861 [cs.CV].

Execution time in milliseconds (GPT-3 is executed on a Tesla A100 GPU)

Workloads	MindSpore	T\/M		AKC	AKC PANAMERA	Improvement over				number of
WORKIDads	windspore	1 1 11	Ansor	ANG	IANAMERA	MindSpore	TVM	Ansor	AKG	fused ops
BERT ^[1]	352.2	138.0	120.3	124.0	111.0	+217%	+24%	+8%	+12%	304
Wide&Deep ^[2]	22.4	12.5	12.8	12.6	11.0	+104%	+14%	+16%	+15%	74
VGG-16 ^[3]	70.4	65.7	66.3	67.6	64.2	+10%	+2%	+3%	+5%	39
MobileNet-v3 ^[4]	151.4	133.0	129.4	136.8	131.5	+15%	+1%	-2%	+4%	52
Transformer ^[5]	157.8	132.4	126.5	136.8	79.2	+99%	+67%	+60%	+73%	746
GPT-3 ^[6]	483.0	133.9	131.3	146.2	123.7	+290%	+8%	+6%	+18%	409
average						+122.5%	+19.3%	+15.2%	+21.2%	

- number of operators fused by PANAMERA in a workload
- PANAMERA enhances the performance of AKG by 21.2% on average

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- $\bullet~{\rm PANAMERA}$ enhances the performance of AKG by 21.2% on average
- AKG + PANAMERA outperforms MindSpore (backed by CUDA libraries), TVM and Ansor by 122.5%, 19.3% and 15.2%, respectively

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- PANAMERA enables fusion of independent reductions, further improving the fusion possibilities and validating that there still exists space for optimizing reductions.

(B)

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MNK shape	K-dim config	PANAMERA	tensor cores	matching percent
$128 \times 32 \times 64$	2 blocks	24.044	4.381	18.22%
128 imes 32 imes 1024	16 blocks	21.378	57.882	270.75%
$1024 \times 512 \times 1024$	16 blocks	183.18	78.623	42.92%

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- non-determinism issue of atomic instructions; the hardware scheme for deterministic atomic buffering^[1] is a solution
- (slight) manual effort required to configure templated routines in the generated code; fully automation is under construction

^[1]Yuan Hsi Chou et al. "Deterministic Atomic Buffering". MICRO 2020, pp. 981–995. ト (ヨト (ヨト (ヨト)) モート ション モート (ヨート) (ロート)

Thank you!



Any Questions?

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