A polyhedral compilation framework for loops with dynamic data-dependent bounds

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Outline

1. Introduction
   - Motivation
   - Examples
2. Polyhedral compilation of dynamic counted loops
   - Schedule tree
   - Program analysis
   - Code generation
3. Experimental results
   - HOG descriptor
   - SpMV computations
   - Inspector-executor codes
4. Conclusion

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Motivation

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`Counted loops` with `dynamic data-dependent bounds` are regular counted loops with numerical constant strides, whose lower and/or upper bound may not be an affine function of enclosing loop counters and loop-invariant parameters.

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```
   for (i=0; i<N; i++) {
S0:  m = condition; // dynamically computed upper bound
     for (j=0; j<m; j++)
S1:    S;
   }
```

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- `dynamic counted loops` are less expressive than general `while` loops.
- Less expressive/general control flow enables more aggressive optimizations.
- Building on the state of the art polyhedral optimization of `while` loops by Benabderrahmane et al. [BPCB10], but the authors' efficient code generation algorithm is not completely described.
- [BPCB10] is constrained by inductive dependences on exit conditions which limit affine transformations and parallelization.

An illustrative example

```
   for (i=0; i<N; i++) {
S0:  condition = ...;
     while (condition) {
S1:    condition = ...;
S2:    S;
     }
   }
```

A general while loop

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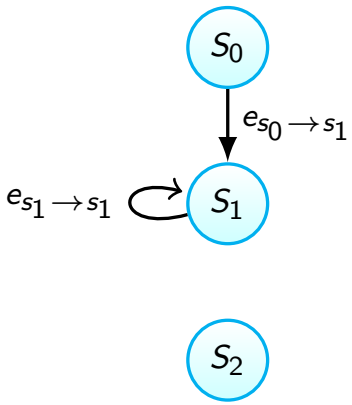

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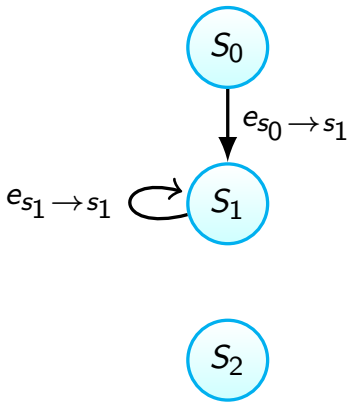


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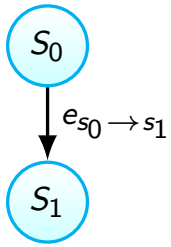

More examples

```
   for (i=0; i<N; i++) {
S0:   m = f(i);
S1:   n = g(i);
      for (j=0; j<m; j++)
        for (k=0; k<n; k++)
S2:       S(i, j, k);
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```

Histogram of Oriented Gradients (HOG)

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   for (i=0; i<N; i++) {
S0:   m = f(i);
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Sparse matrix-vector CSR

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   for (k=0; k<2*M-1; k++) {
S0:   m = f(k);
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Sparse matrix-vector DIA

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Sparse matrix-vector ELL

CSR: compressed sparse row DIA: diagonal ELL: ELLPack

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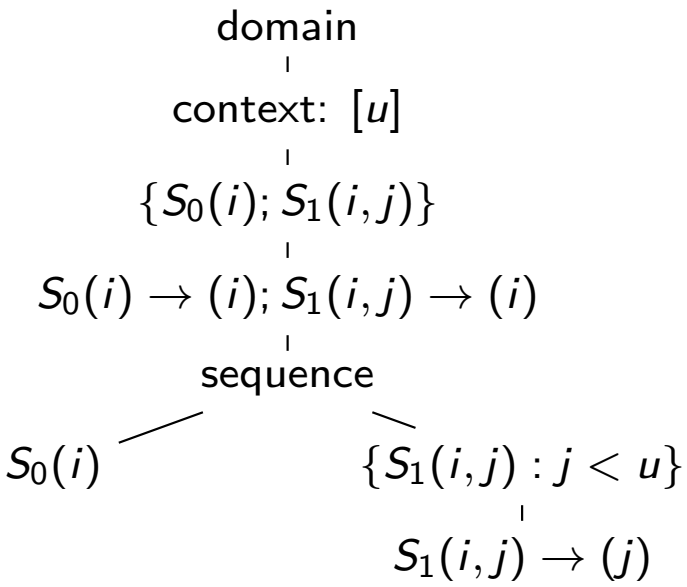
Schedule tree (in isl)



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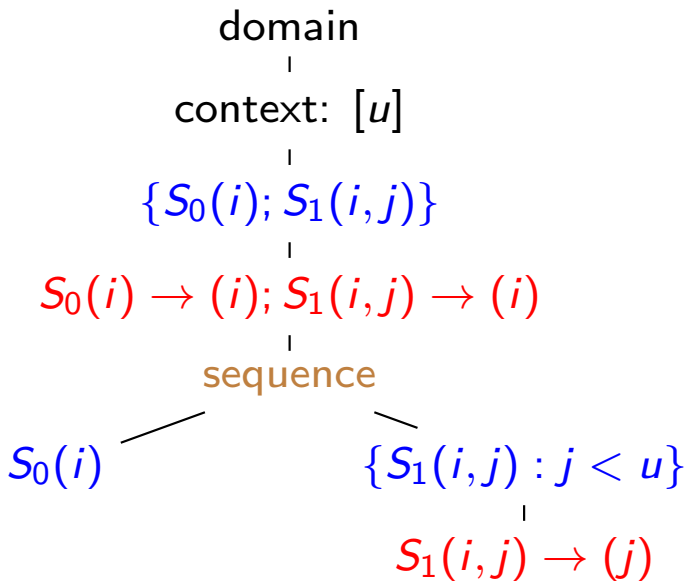

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  - Band: multi-dimensional piecewise quasi-affine partial schedule
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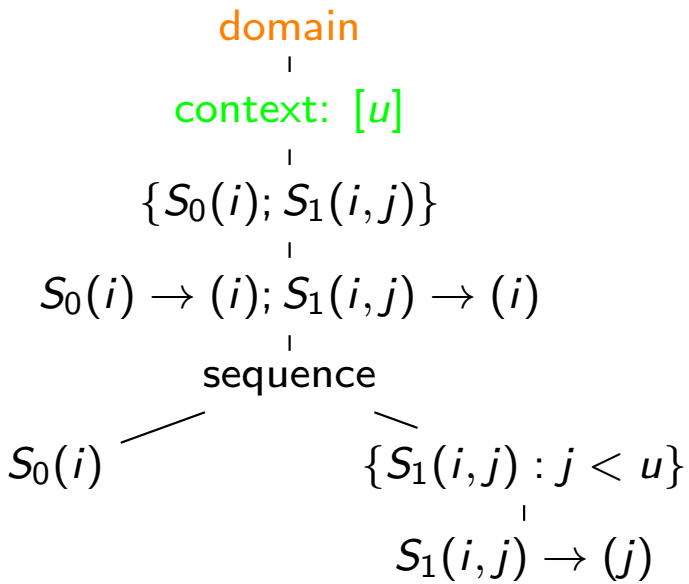

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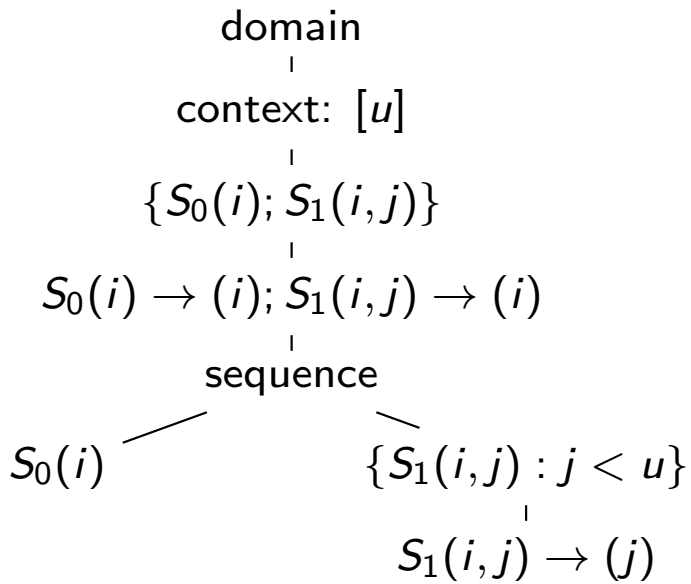

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- Convenience node types
  - Mark: attach additional information to subtrees

Program analysis

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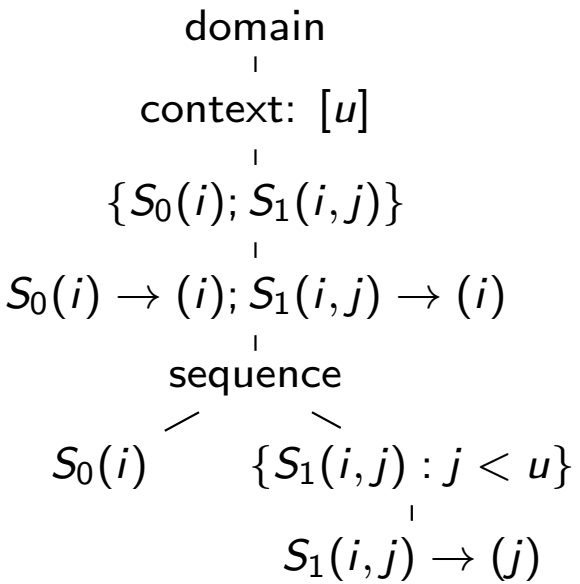

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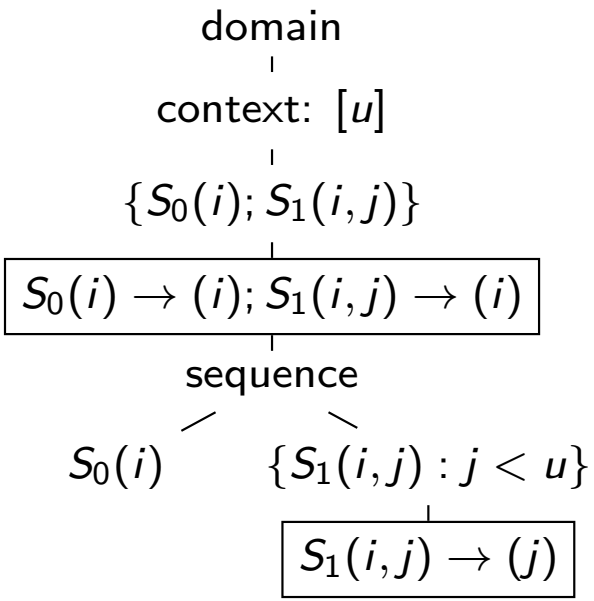

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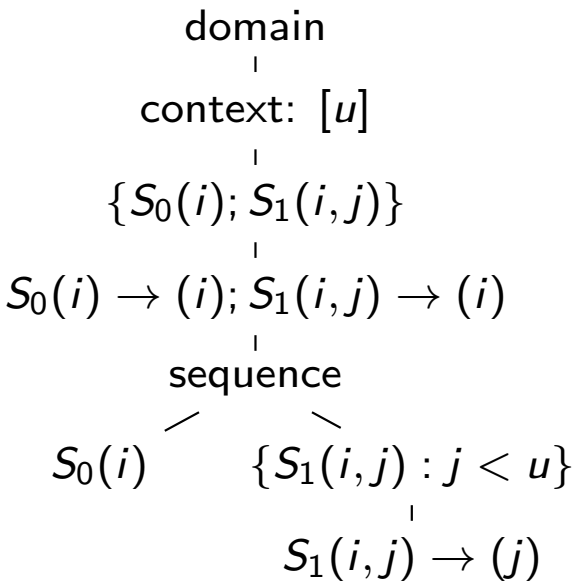

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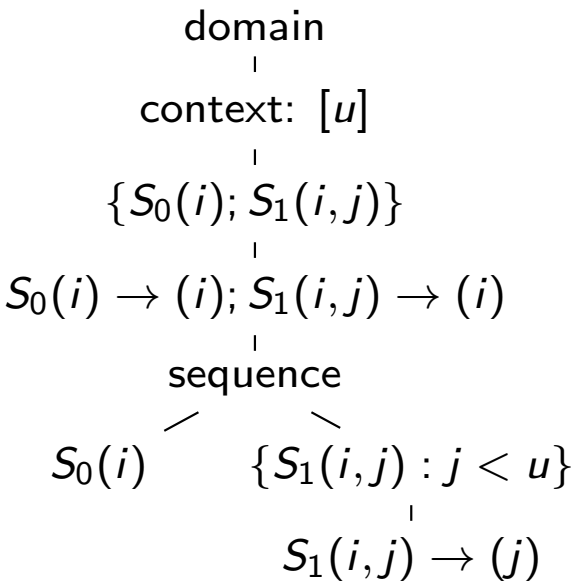

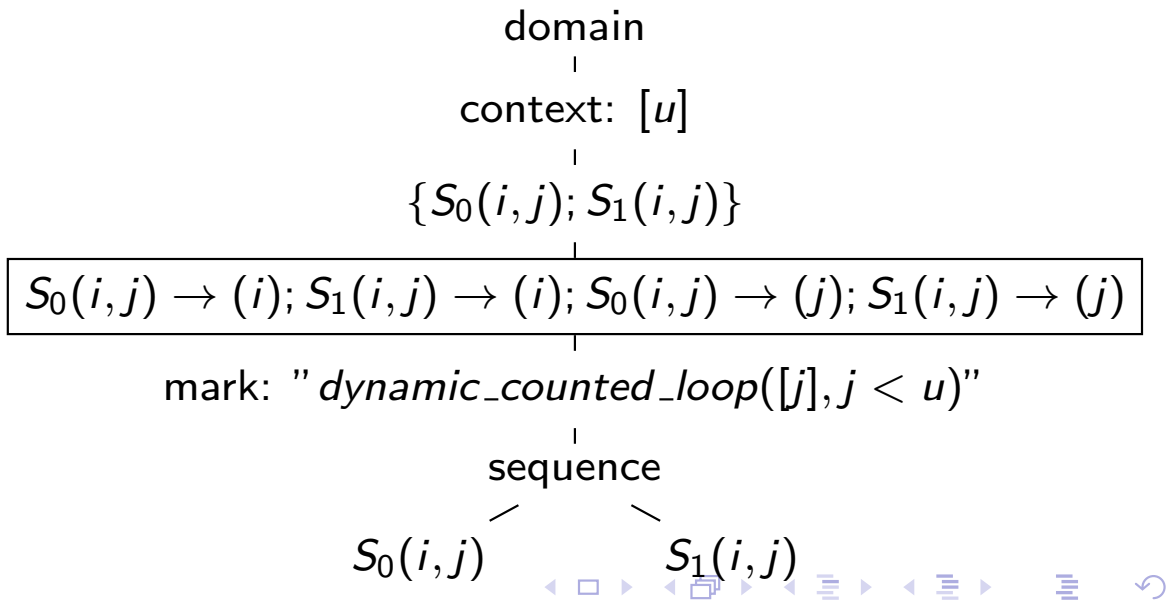

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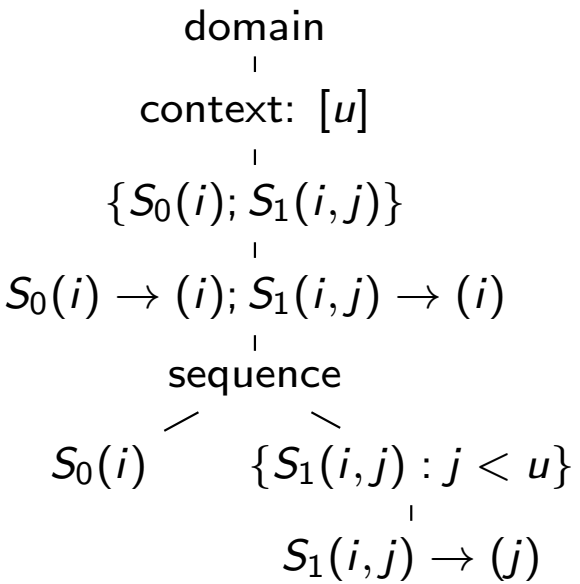

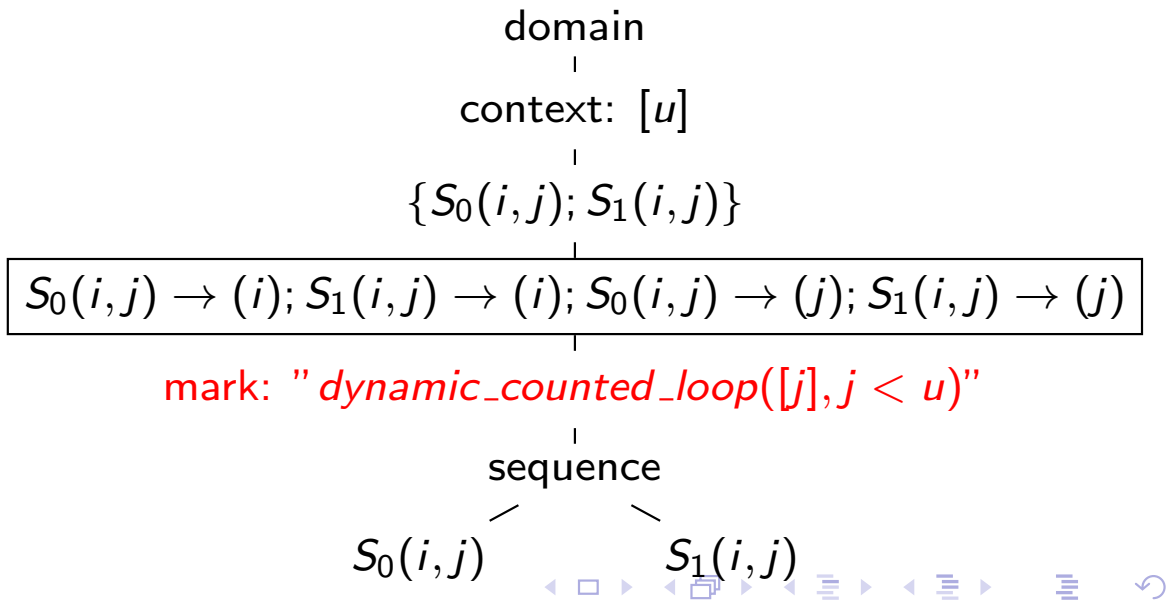

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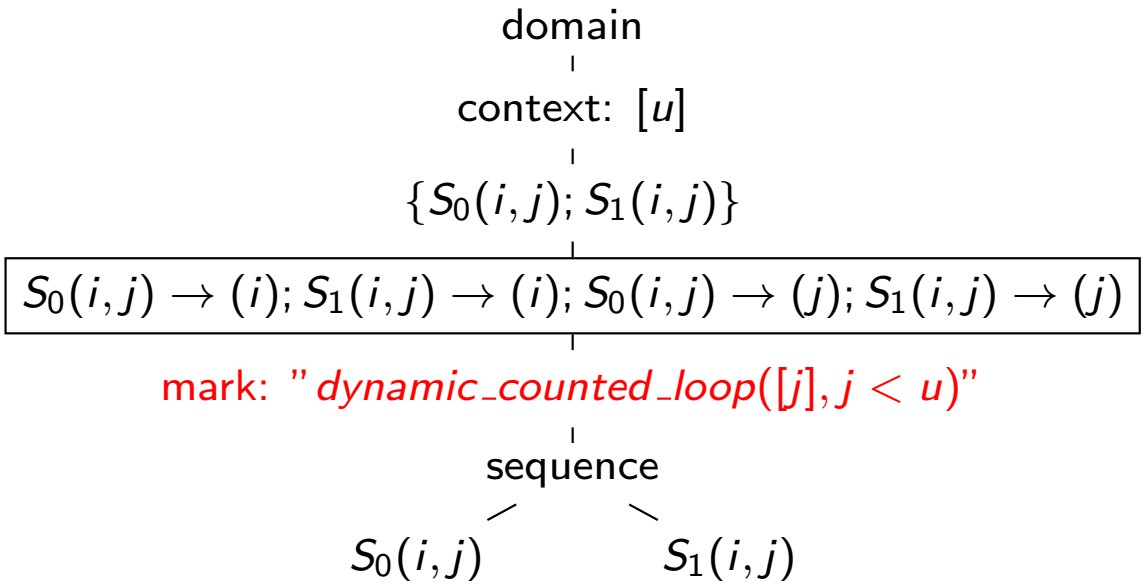

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The code generation template

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   for (i=0; i<N; i++)
     for (j=0; j<u1; j++) {
       for (k=0; k<u2; k++) {
         for (...) {
S0:        m = f(i);
S1:        n = g(i);
             ...
Sn:        if (j<m&&k<n&&...)
             S(i, j, k, ...);
           ...
         }
         if (k>=n)
           break;
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SpMV CSR code

```
 for (i=0; i<N; i++)
   for (j=0; j<u; j++) {
S0:  m = idx[i+1] - idx[i];
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```
   for (ii=32*b0; ii<N; ii+=8192)
     for (jj=32*b1; jj<u; jj+=8192) {
       for (i=t0; i<=min(31,N-ii); i+=32)
         for(j=t1; i<=min(31,u-jj); i+=32) {
S0:        m = idx[ii+i+1] - idx[ii+i];
S1:        if (jj+j<m)
             y[ii+i] += A[jj+j]*x[col[jj+j]];
           if (jj+j>=m)
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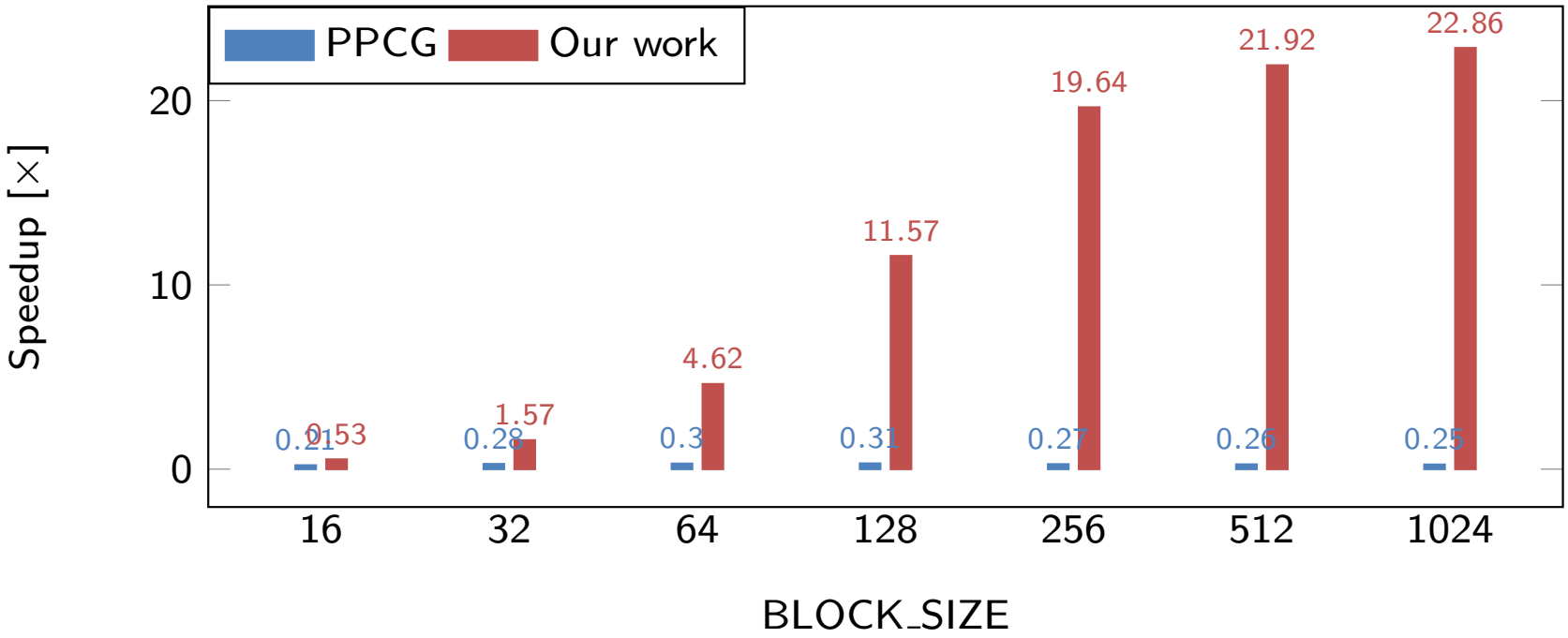
HOG descriptor

Performance of the HOG descriptor including data transfer time[1]



[1] Running on NVIDIA Quadro K4000 GPU with Intel Xeon E5-2630 CPU

HOG descriptor

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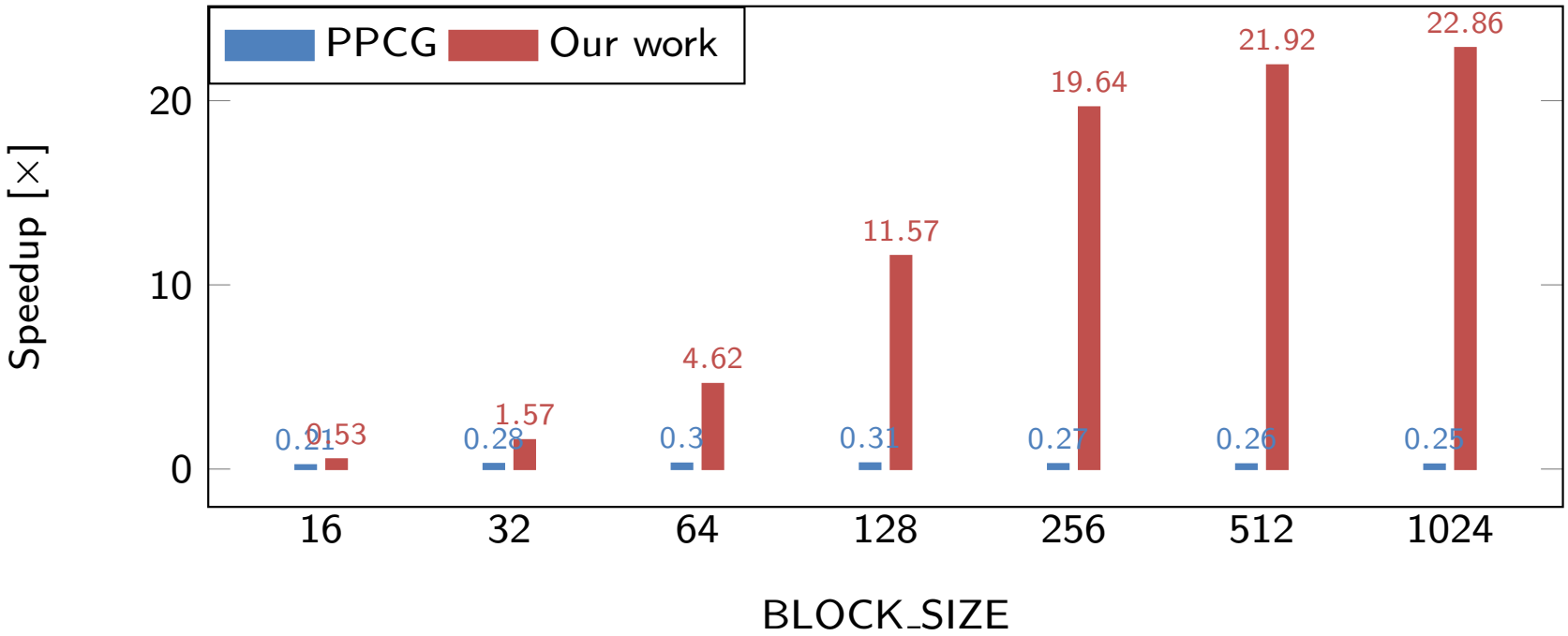
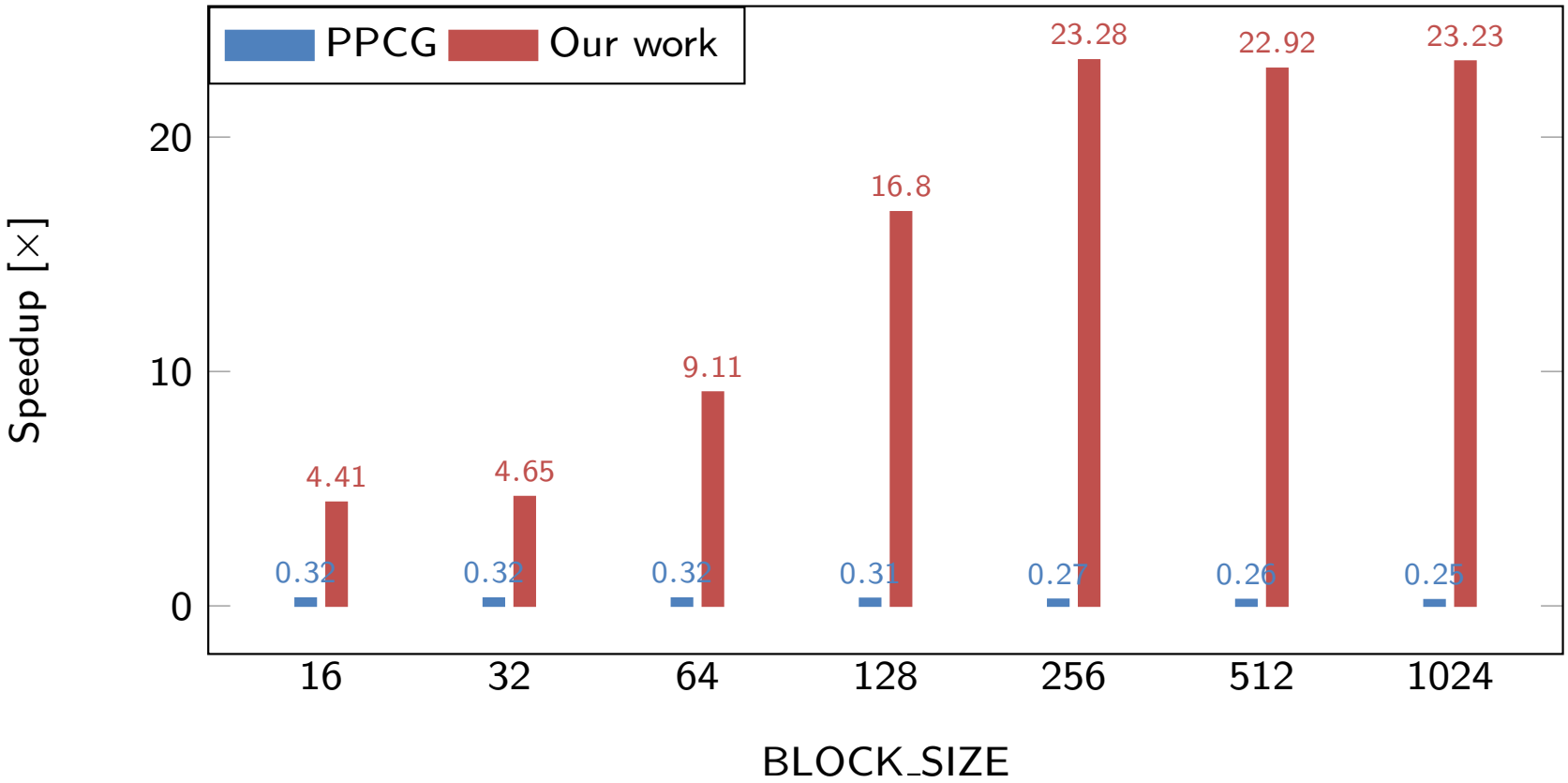


Our work enables the effective parallelization and optimization of HOG on the target platform, outperforming sequential execution and PPCG's inner parallelism version (total execution time including data transfers)

[1] Running on NVIDIA Quadro K4000 GPU with Intel Xeon E5-2630 CPU

HOG descriptor

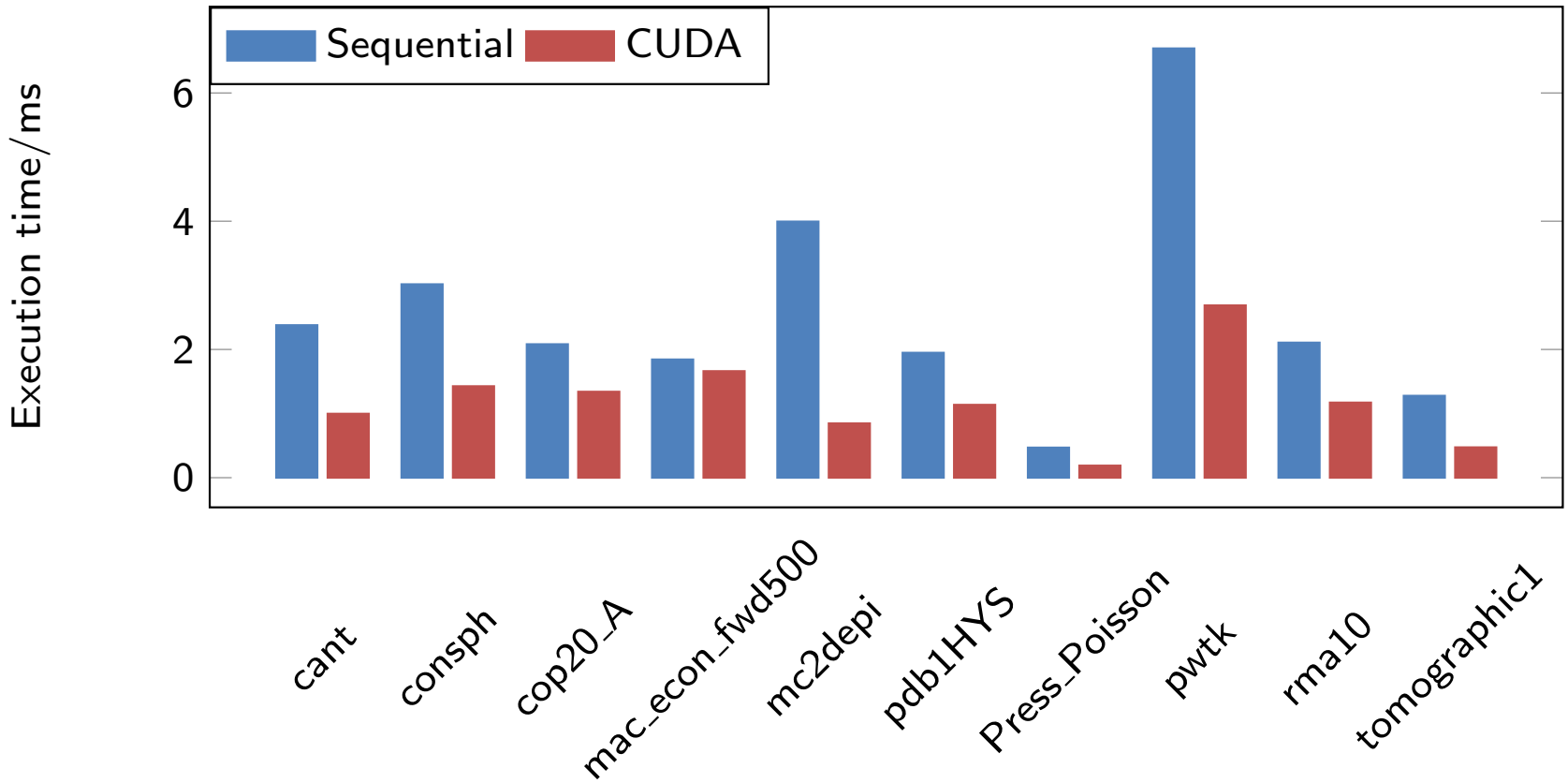
Performance of the HOG descriptor without data transfer time[2]



[2]The performance is the speedup w.r.t. the sequential execution time

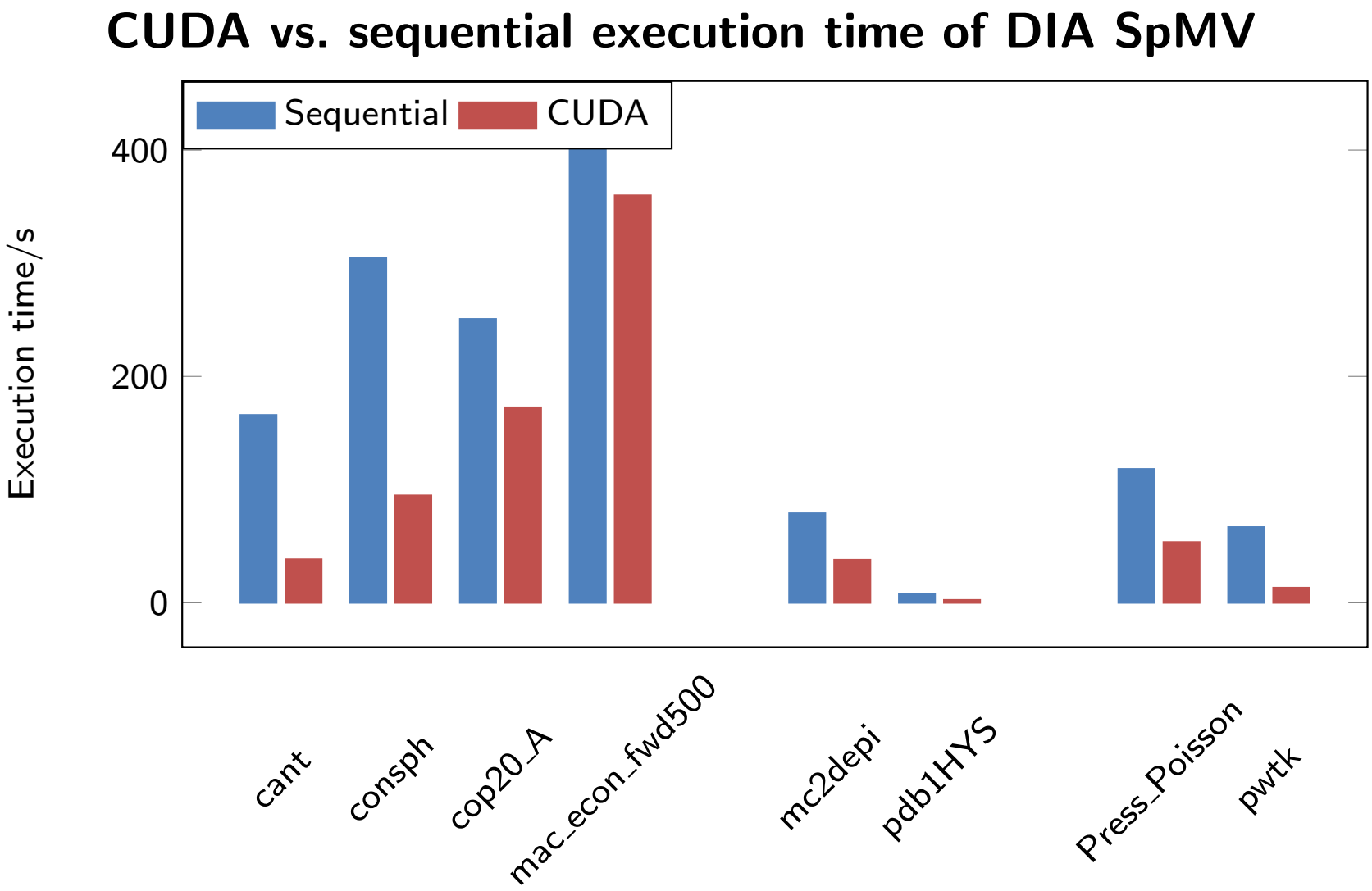
SpMV computations

CUDA vs. sequential execution time of CSR SpMV[3]

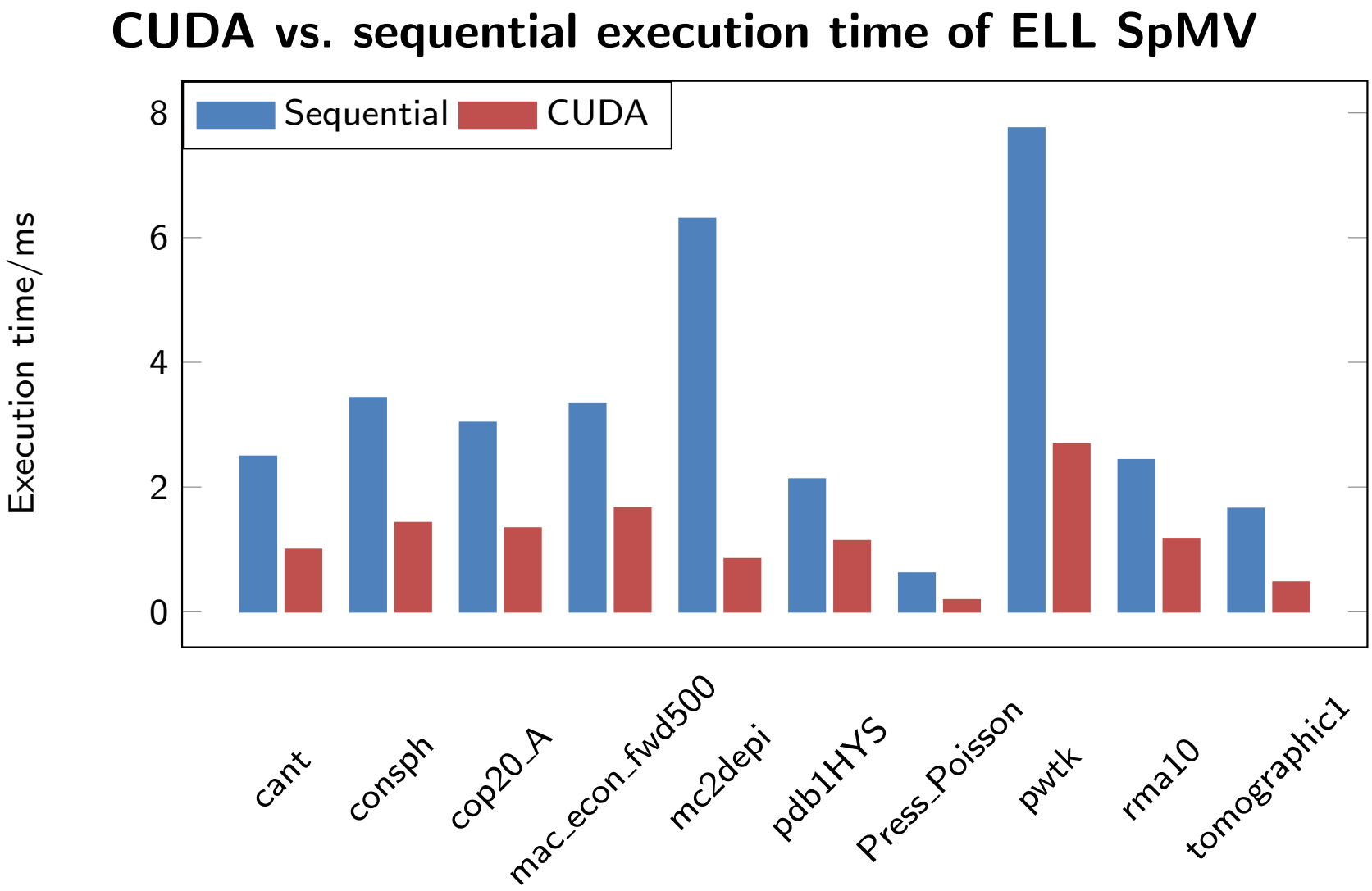


[3] Based on the University of Florida sparse matrix collection [DH11]

SpMV computations



SpMV computations



Inspector-executor codes

An inspector-executor implementation of CSR SpMV

```
for (i=0; i<M; i++) {
  for (k=0; k<N; k++) {
    marked = false;
    for (j=idx[i]; j<idx[i+1]; j++)
      if (k == col[j])
        if (!marked) {
          marked = true;
          exp_idx[count] = k;
          count++;
        }
  }
  f_idx[i+1] = count;
}
```

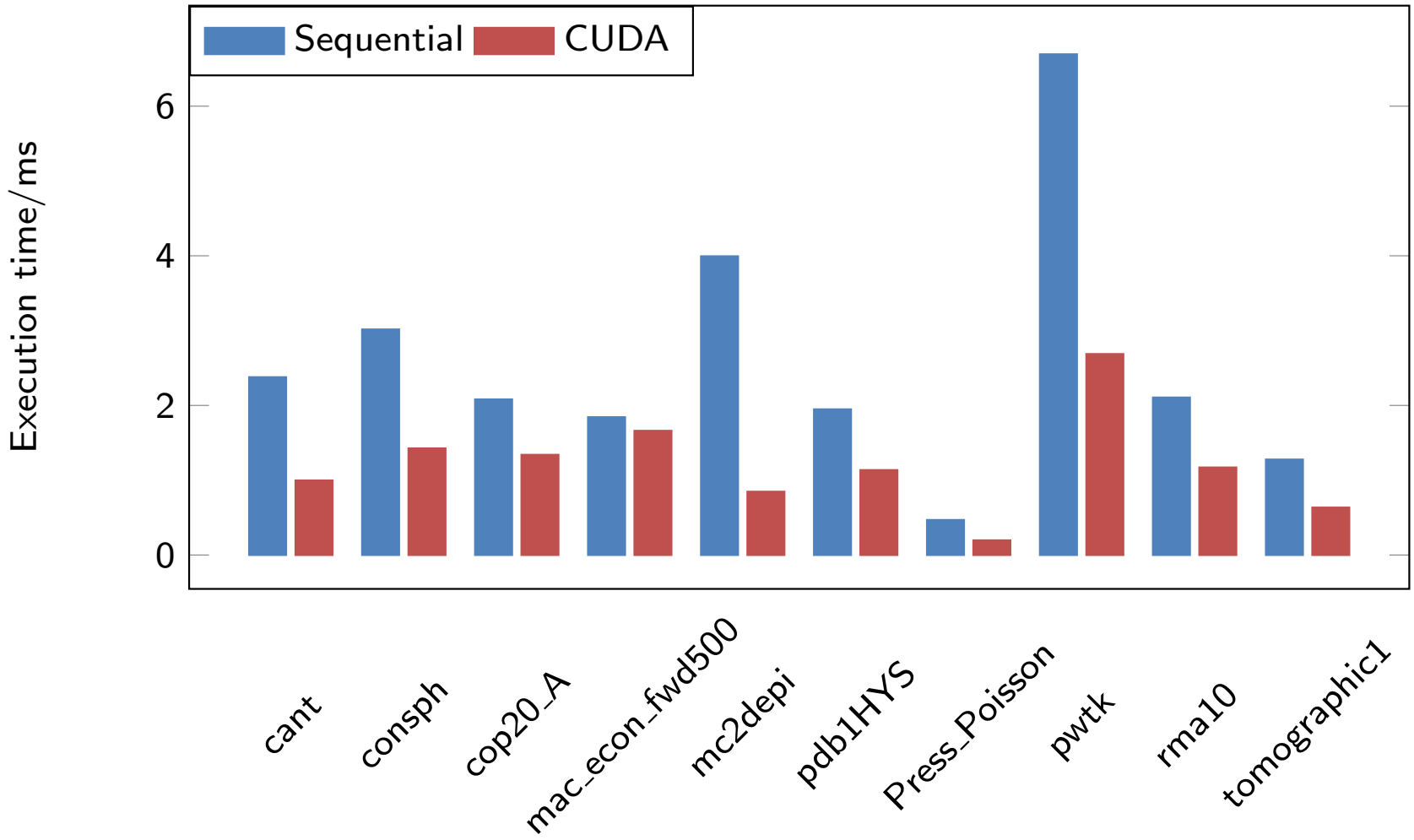
inspector

```
for (i=0; i<N; i++) {
  m = f_idx[i+1] - f_idx[i];
  for (j=0; j<m; j++)
    if (j<m)
      y[i] += val[j+f_idx[i]] * x[exp_idx[j+f_idx[i]]];
}
```

executor

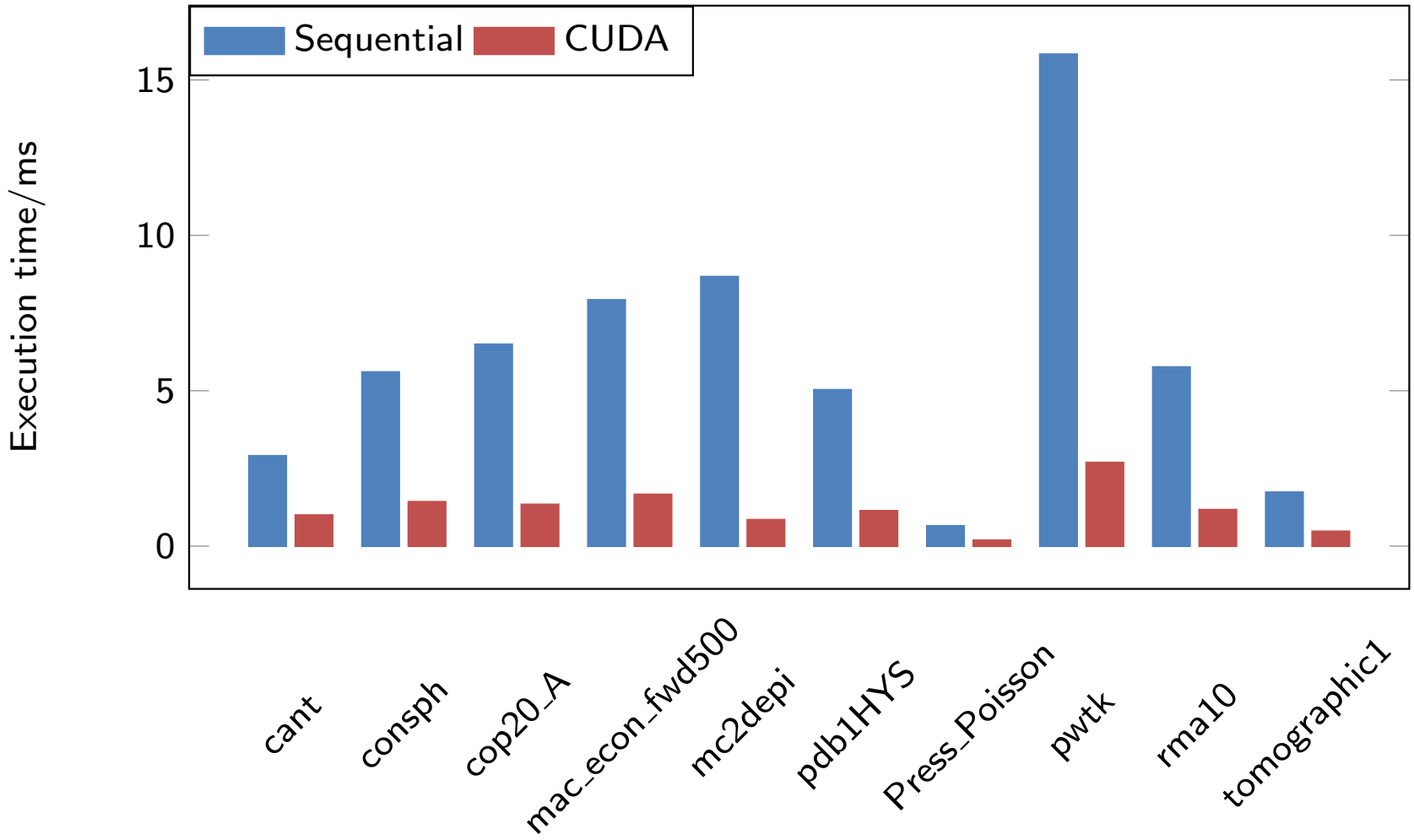
Inspector-executor codes

CUDA vs. sequential execution time of the CSR SpMV executor



Inspector-executor codes

CUDA vs. sequential execution time of the ELL SpMV executor



Outline

1. Introduction
   - Motivation
   - Examples
2. Polyhedral compilation of dynamic counted loops
   - Schedule tree
   - Program analysis
   - Code generation
3. Experimental results
   - HOG descriptor
   - SpMV computations
   - Inspector-executor codes
4. Conclusion

Conclusion

- Our work

- Future work

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  - models control dependences on data-dependent predicates by revisiting the work of Benabderrahmane et al. [BPCB10].
  - does not resort to more expressive first-order logic with non-interpreted functions/predicates, like [SCF03, SLC+16].
  - provides code generation templates for multiple scenarios, including the inspector-executor scheme [VHS15].
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  - models control dependences on data-dependent predicates by revisiting the work of Benabderrahmane et al. [BPCB10].
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- Future work
  - fully automate and implement the framework in PPCG [VCJC+13].
  - conduct further experiments on CPU and GPU platforms, comparing the performance with the CUSP library.

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